Reviewing non-survey approaches for constructing interregional IO tables

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Content

Introduction

Starting point:

- Regional economic models require adequate modeling of **intra- and interregional** IO transactions
- **Regionalization** of national IO tables is a standard approach for constructing corresponding tables
- The type of national table used for regionalization depends on the **goal of the analysis**
- The type of national table determines the type of interregional table

This presentation: Review of types of tables and corresponding regionalization approaches

Content

Type B national IO table

Tabelle: Type B national IO table

sector 1	...	sector N			
z_{11}	$\dots$	z_{1N}	y_1	e_1	x_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
z_{N1}	$\dots$	z_{NN}	y_N	e_N	x_N
m_1	$\dots$	m_N			
v_1	$\dots$	v_N			
x_1	$\dots$	x_N			

- IO matrix z_{ij} considers domestic activity only

Type B national IO table

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sector 1	…	sector N			
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$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
z_{N1}	…	z_{NN}	y_N	e_N	x_N
m_1	…	m_N			
v_1	…	v_N			
x_1	…	x_N			

- IO matrix z_{ij} considers domestic activity only
- Value added v_i, final demand y_i, exports e_i

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z_{N1}	...	z_{NN}	y_N	e_N	x_N
m_1	...	m_N			
v_1	...	v_N			
x_1	...	x_N			

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m_1	...	m_N			
v_1	...	v_N			
x_1	...	x_N			

- IO matrix z_{ij} considers domestic activity only
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- Import matrix M_{ji} (product by sector) often provided

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z_{N1}	...	z_{NN}	y_N	e_N	x_N
m_1	...	m_N			
v_1	...	v_N			
x_1	...	x_N			

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- Value added v_i, final demand y_i, exports e_i
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- Import matrix M_{ji} (product by sector) often provided
- Column and row totals refer to output x_i

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$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
z_{N1}	...	z_{NN}	y_N	e_N	x_N
m_1	...	m_N			
v_1	...	v_N			
x_1	...	x_N			

- IO matrix z_{ij} considers domestic activity only
- Value added v_i, final demand y_i, exports e_i
- Row vector of imports $m_i = \sum_j M_{ji}$ (sum over products)
- Import matrix M_{ji} (product by sector) often provided
- Column and row totals refer to output x_i
- Useful to analyze **domestic** activity

Type A national table

Tabelle: Type A national IO table

sector 1	...	sector N				
$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	$-\bar{m}_1$	x_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	$-\bar{m}_N$	x_N
v_1	...	v_N				
x_1	...	x_N				

- IO matrix $\bar{z}_{ij} = z_{ij} + M_{ij}$ considers domestic and foreign products

Type A national table

Tabelle: Type A national IO table

sector 1	...	sector N				
$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	$-\bar{m}_1$	x_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	$-\bar{m}_N$	x_N
v_1	...	v_N				
x_1	...	x_N				

- IO matrix $\bar{z}_{ij} = z_{ij} + M_{ij}$ considers domestic and foreign products
- Column vector of imports has to be subtracted for consistency $\bar{m}_i = \sum_j M_{ij}$ (sum over sectors!)

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$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	$-\bar{m}_1$	x_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	$-\bar{m}_N$	x_N
v_1	...	v_N				
x_1	...	x_N				

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$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	$-\bar{m}_1$	x_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	$-\bar{m}_N$	x_N
v_1	...	v_N				
x_1	...	x_N				

- IO matrix $\bar{z}_{ij} = z_{ij} + M_{ij}$ considers domestic and foreign products
- Column vector of imports has to be subtracted for consistency $\bar{m}_i = \sum_j M_{ij}$ (sum over sectors!)
- Column and row totals refer to output x_i
- Useful to analyze **global** effects such as footprints

Type E national table

Tabelle: Type E national IO table

sector 1	...	sector N			
$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	u_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	u_N
v_1	...	v_N			
$\bar{m}_1$	...	$\bar{m}_N$			
u_1	...	u_N			

- IO matrix $\bar{z}_{ij} = z_{ij} + M_{ij}$ considers domestic and foreign products and is equivalent to type A

Type E national table

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sector 1	...	sector N			
$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	u_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	u_N
v_1	...	v_N			
$\bar{m}_1$	...	$\bar{m}_N$			
u_1	...	u_N			

- IO matrix $\bar{z}_{ij} = z_{ij} + M_{ij}$ considers domestic and foreign products and is equivalent to type A
- Row vector of imports $\bar{m}_i = \sum_j M_{ij}$ (transposed)

Type E national table

Tabelle: Type E national IO table

sector 1	...	sector N			
$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	u_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	u_N
v_1	...	v_N			
$\bar{m}_1$	...	$\bar{m}_N$			
u_1	...	u_N			

- IO matrix $\bar{z}_{ij} = z_{ij} + M_{ij}$ considers domestic and foreign products and is equivalent to type A
- Row vector of imports $\bar{m}_i = \sum_j M_{ij}$ (transposed)
- Column and row totals refer to total use of product u_i

Type E national table

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sector 1	...	sector N			
$\bar{z}_{11}$	...	$\bar{z}_{1N}$	y_1	e_1	u_1
$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{z}_{N1}$	...	$\bar{z}_{NN}$	y_N	e_N	u_N
v_1	...	v_N			
$\bar{m}_1$	...	$\bar{m}_N$			
u_1	...	u_N			

- IO matrix $\bar{z}_{ij} = z_{ij} + M_{ij}$ considers domestic and foreign products and is equivalent to type A
- Row vector of imports $\bar{m}_i = \sum_j M_{ij}$ (transposed)
- Column and row totals refer to total use of product u_i
- More appealing for regionalization than type A because of **non-negative** entries

Regionalization of IO tables

Goal: Estimation of IO relations within and between regions from national IO table

Feasibility constraints:

- **Aggregation** constraint: Regional data must add up to known national data
- **Balance** constraint: Row sum equal to column sum for each regional sector/product.

Content

Type B interregional IO table

	region 1		$\cdots$		region R				
z^{11}_{11}	$\cdots$	z^{11}_{1N}	$\cdots$	z^{1R}_{11}	$\cdots$	z^{1R}_{1N}	y^{1}_{1}	e^{1}_{1}	x^{1}_{1}
$\vdots$	$\ddots$	$\vdots$		$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
z^{11}_{N1}	$\cdots$	z^{11}_{NN}	$\cdots$	z^{1R}_{N1}	$\cdots$	z^{1R}_{NN}	y^{1}_{N}	e^{1}_{N}	x^{1}_{N}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
z^{R1}_{11}	$\cdots$	z^{R1}_{1N}	$\cdots$	z^{RR}_{11}	$\cdots$	z^{RR}_{1N}	y^{R}_{1}	e^{R}_{1}	x^{R}_{1}
$\vdots$	$\ddots$	$\vdots$		$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
z^{R1}_{N1}	$\cdots$	z^{R1}_{NN}	$\cdots$	z^{RR}_{N1}	$\cdots$	z^{RR}_{NN}	y^{R}_{N}	e^{R}_{N}	x^{R}_{N}
m^{1}_{1}	$\cdots$	m^{1}_{N}	$\cdots$	m^{R}_{1}	$\cdots$	m^{R}_{N}			
v^{1}_{1}	$\cdots$	v^{1}_{N}	$\cdots$	v^{R}_{1}	$\cdots$	v^{R}_{N}			
x^{1}_{1}	$\cdots$	x^{1}_{N}	$\cdots$	x^{R}_{1}	$\cdots$	x^{R}_{N}			

- Notation: $z^{s \to r}_{i \to j}$ transactions from sector i in s to sector j in r
- Blocks of IO matrices for each pair of regions, intraregional matrices on the diagonal
- Imports, value added, final demand and exports are vectors analogous to the type B national table

Steps in type B table regionalization

General framework for regionalizing **type B** national IO tables:

1. Intraregional input coefficients estimated by **location quotients**
2. Interregional input coefficients estimated by gravity models
3. Imports, value, added, final demand and exports estimated from available data
4. Application of numerical algorithm to reach feasibility

Type B table regionalization: Step 1

Estimation of **intra**regional transactions (blocks on the diagonal):

- National input coefficients $a_{ij} = {}^{z_{ij}}/_{x_j}$

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- National input coefficients $a_{ij} = {}^{z_{ij}}/_{x_j}$
- Measure of regional activity ϵ_i^r (output, value added, employment, . . .)

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- Regional output x_i^r (row/column total) estimated from ϵ_i^r such that $\sum_r x_i^r = x_i$

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- Location-quotient LQ_{ij}^r non-negative and increasing in ϵ_i^r

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Estimation of **intra**regional transactions (blocks on the diagonal):

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- Location-quotient LQ_{ij}^r non-negative and increasing in ϵ_i^r
- Estimated intraregional input coefficients $a_{ij}^{rr} = min(a_{ij}, a_{ij} \cdot LQ_{ij}^r)$

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- National input coefficients $a_{ij} = {}^{z_{ij}}/_{x_j}$
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Type B table regionalization: Step 1

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- National input coefficients $a_{ij} = z_{ij}/x_j$
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- Location-quotient LQ_{ij}^r non-negative and increasing in ϵ_i^r
- Estimated intraregional input coefficients $a_{ij}^{rr} = min(a_{ij}, a_{ij} \cdot LQ_{ij}^r)$
- Estimated intraregional transactions $z_{ij}^{rr} = a_{ij}^{rr} \cdot x_j^r$
- $\sum_r z_{ij}^{rr} \leq z_{ij}$ for all (i, j) follows from $a_{ij}^{rr} \leq a_{ij}$ and leaves room for interregional transactions $z_{ij}^{sr} (s \neq r)$

Type B table regionalization: Step 2

Estimation of **inter**regional transactions (off-diagonal blocks)

- Gravity model: $\ln(z_{ij}^{sr}) = c + \beta_1 \ln(d_{sr}) + \beta_2 \ln(x_i^s) + \beta_3 \ln(x_j^r)$

Type B table regionalization: Step 2

Estimation of **inter**regional transactions (off-diagonal blocks)

- Gravity model: $\ln(z_{ij}^{sr}) = c + \beta_1 \ln(d_{sr}) + \beta_2 \ln(x_i^s) + \beta_3 \ln(x_j^r)$
- Geographical distance between regions d_{sr}

Type B table regionalization: Step 2

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- Since true transactions are unknown, β_k must come from the literature or other analyses

Type B table regionalization: Step 2

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- Geographical distance between regions d_{sr}
- Since true transactions are unknown, β_k must come from the literature or other analyses
- Credible values: $-1.5 \leq \beta_1 \leq -0.5$; $0.8 \leq \beta_2, \beta_3 \leq 1$

Type B table regionalization: Step 2

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- Credible values: $-1.5 \leq \beta_1 \leq -0.5$; $0.8 \leq \beta_2, \beta_3 \leq 1$
- Estimated interregional transactions $z_{ij}^{sr} = d_{sr}^{\beta_1} \cdot (x_i^s)^{\beta_2} \cdot (x_j^r)^{\beta_3}$

Type B table regionalization: Step 2

Estimation of **inter**regional transactions (off-diagonal blocks)

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- Geographical distance between regions d_{sr}
- Since true transactions are unknown, β_k must come from the literature or other analyses
- Credible values: $-1.5 \leq \beta_1 \leq -0.5$; $0.8 \leq \beta_2, \beta_3 \leq 1$
- Estimated interregional transactions $z_{ij}^{sr} = d_{sr}^{\beta_1} \cdot (x_i^s)^{\beta_2} \cdot (x_j^r)^{\beta_3}$
- Scaling such that $\sum_{s,r} z_{ij}^{sr} = z_{ij}$ for all (i, j), using the already derived intraregional transaction estimates

Type B table regionalization: Step 3

Estimation of additional components of IO table:

- Regional (foreign) trade data might be available to estimate imports m_i^r and exports e_i^r
- Note that m_i^r refers to intermediate inputs used by domestic sectors and does not include direct imports by households (such as tourism)
- Value added v_i^r and final demand y_i^r proportional to output x_i^r

Type B table regionalization: Step 4

Balancing algorithm:

- Idea: Turning previously derived estimates z_{ij}^{sr}, m_i^r, v_i^r, y_i^r and e_i^r into feasible estimates (without changing them too much)
- Solution: Constrained optimization (minimizing quadratic deviation, maximizing entropy, . . .)

Type B table regionalization: Step 4

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Example for optimization problem with known x_i^r:

$$\text{Minimize } S = \sum \frac{(\hat{z}_{ij}^{sr} - z_{ij}^{sr})^2}{\hat{z}_{ij}^{sr}} + \sum \frac{(\hat{m}_i^r - m_i^r)^2}{\hat{m}_i^r}$$
$$+ \sum \frac{(\hat{v}_i^r - v_i^r)^2}{\hat{v}_i^r} + \sum \frac{(\hat{y}_i^r - y_i^r)^2}{\hat{y}_i^r} + \sum \frac{(\hat{e}_i^r - e_i^r)^2}{\hat{e}_i^r}$$

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Example for optimization problem with known x_i^r:

$$\text{Minimize } S = \sum \frac{(\hat{z}_{ij}^{sr} - z_{ij}^{sr})^2}{\hat{z}_{ij}^{sr}} + \sum \frac{(\hat{m}_i^r - m_i^r)^2}{\hat{m}_i^r}$$
$$+ \sum \frac{(\hat{v}_i^r - v_i^r)^2}{\hat{v}_i^r} + \sum \frac{(\hat{y}_i^r - y_i^r)^2}{\hat{y}_i^r} + \sum \frac{(\hat{e}_i^r - e_i^r)^2}{\hat{e}_i^r}$$

subject to:

$\sum_{j,s} \hat{z}_{ji}^{sr} + \hat{m}_i^r + \hat{v}_i^r = x_i^r$ (balance constraint)

$\sum_{j,s} \hat{z}_{ij}^{rs} + \hat{y}_i^r + \hat{e}_i^r = x_i^r$ (balance constraint)

$\sum_{s,r} \hat{z}_{ij}^{sr} = z_{ij}$; $\sum_r \hat{m}_i^r = m_i$; $\sum_r \hat{v}_i^r = v_i$; $\sum_r \hat{y}_i^r = y_i$; $\sum_r \hat{e}_i^r = e_i$ (aggregation constraints)

Content

Type E interregional IO table

region 1			$\cdots$	region R				exports to other regions					
$\bar{z}^{11}_{11}$	$\cdots$	$\bar{z}^{11}_{1N}$					y^1_1	0	t^{12}_1	$\cdots$	t^{1R}_1	e^1_1	u^1_1
$\vdots$	$\ddots$	$\vdots$					$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
$\bar{z}^{11}_{N1}$	$\cdots$	$\bar{z}^{11}_{NN}$					y^1_N	0	t^{12}_N	$\cdots$	t^{1R}_N	e^1_N	u^1_N
			$\ddots$				$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
				$\bar{z}^{RR}_{11}$	$\cdots$	$\bar{z}^{RR}_{1N}$	y^R_1	t^{R1}_1	t^{R2}_1	$\cdots$	0	e^R_1	u^R_1
				$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
				$\bar{z}^{RR}_{N1}$	$\cdots$	$\bar{z}^{RR}_{NN}$	y^R_N	t^{R1}_N	t^{R2}_1	$\cdots$	0	e^R_N	u^R_N
v^1_1	$\cdots$	v^1_N	$\cdots$	v^R_1	$\cdots$	v^R_N							
0	$\cdots$	0	$\cdots$	t^{1R}_1	$\cdots$	t^{1R}_N							
t^{21}_1	$\cdots$	t^{21}_N	$\cdots$	t^{2R}_1	$\cdots$	t^{2R}_N							
$\vdots$		$\vdots$		$\vdots$		$\vdots$							
t^{R1}_1	$\cdots$	t^{R1}_N	$\cdots$	0	$\cdots$	0							
$\bar{m}^1_1$	$\cdots$	$\bar{m}^1_N$	$\cdots$	$\bar{m}^R_1$	$\cdots$	$\bar{m}^R_N$							
u^1_1	$\cdots$	u^1_N	$\cdots$	u^R_1	$\cdots$	u^R_N							

- IO matrices include imports from other regions and abroad
- Interregional transactions treated as exports/imports
- More columns and rows than type B interregional table

Type E table regionalization

General framework for regionalizing **type E** national IO tables:

1. Intraregional input coefficients equal to national coefficients (technical coefficients)
2. Regional commodity in- and outflows estimated as aggregates
3. Interregional product flows estimated by gravity model
4. Application of numerical algorithm to reach feasibility

Type E table regionalization: Step 1

Estimation of **intra**regional transactions:

- Regional input coefficients equal to national coefficients $\bar{a}^r_{ij} = \bar{a}_{ij} = {}^{\bar{z}_{ij}}/_{x_j}$
- Therefore, regional input coefficients are (also) technical coefficients and the same in every region
- Resulting in volumes $\bar{z}^r_{ij} = \bar{a}^r_{ij} \cdot x^r_j$

Type E table regionalization: Step 2

Estimation of aggregate regional trade:

- Foreign imports $\bar{m}_i^r$ and foreign exports e_i^r estimated by available data

Type E table regionalization: Step 2

Estimation of aggregate regional trade:

- Foreign imports $\bar{m}_i^r$ and foreign exports e_i^r estimated by available data
- Total use u_i^r (row/column totals) proportional to output x_i^r (latter is assumed to be known)

Type E table regionalization: Step 2

Estimation of aggregate regional trade:

- Foreign imports $\bar{m}_i^r$ and foreign exports e_i^r estimated by available data
- Total use u_i^r (row/column totals) proportional to output x_i^r (latter is assumed to be known)
- Aggregate inflow from other regions $\sum_s t_i^{sr} = u_i^r - x_i^r - \bar{m}_i^r$ (from supply side)

Type E table regionalization: Step 2

Estimation of aggregate regional trade:

- Foreign imports $\bar{m}_i^r$ and foreign exports e_i^r estimated by available data
- Total use u_i^r (row/column totals) proportional to output x_i^r (latter is assumed to be known)
- Aggregate inflow from other regions $\sum_s t_i^{sr} = u_i^r - x_i^r - \bar{m}_i^r$ (from supply side)
- Alternatively: Aggregate outflow to other regions $\sum_s t_i^{rs} = u_i^r - y_i^r - \sum_j \bar{z}_{ij}^r - e_i^r$ (from use side)

Type E table regionalization: Step 2

Estimation of aggregate regional trade:

- Foreign imports $\bar{m}_i^r$ and foreign exports e_i^r estimated by available data
- Total use u_i^r (row/column totals) proportional to output x_i^r (latter is assumed to be known)
- Aggregate inflow from other regions $\sum_s t_i^{sr} = u_i^r - x_i^r - \bar{m}_i^r$ (from supply side)
- Alternatively: Aggregate outflow to other regions $\sum_s t_i^{rs} = u_i^r - y_i^r - \sum_j \bar{z}_{ij}^r - e_i^r$ (from use side)
- Aggregate set to zero if right-hand side is negative

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- Aggregate set to zero if right-hand side is negative
- Value added v_i^r and final demand y_i^r proportional to output x_i^r (as for type B)

Type E table regionalization: Step 3

Estimation of **inter**regional transactions:

- $t_i^{sr} = d_{sr}^{\beta_1} \cdot (x_i^s)^{\beta_2} \cdot (x_i^r)^{\beta_3}$
- Formula independent of supply- or use-side approach (step 2)
- Scaling such that aggregate derived in step 2 is matched

Type E table regionalization: Step 4

Balancing algorithm:
Turning previously derived estimates $\bar{z}_{ij}^r$, t_i^{sr}, u_i^r, $\bar{m}_i^r$, v_i^r, y_i^r and e_i^r into feasible estimates (without changing them too much)

Type E table regionalization: Step 4

Balancing algorithm:
Turning previously derived estimates $\bar{z}_{ij}^r$, t_i^{sr}, u_i^r, $\bar{m}_i^r$, v_i^r, y_i^r and e_i^r into feasible estimates (without changing them too much)

Example for optimization problem with known x_i^r:

$$\text{Minimize } S = \sum \frac{(\hat{\bar{z}}_{ij}^r - \bar{z}_{ij}^r)^2}{\hat{\bar{z}}_{ij}^r} + \sum \frac{(\hat{t}_i^{sr} - t_i^{sr})^2}{\hat{t}_i^{sr}} + \sum \frac{(\hat{u}_i^r - u_i^r)^2}{\hat{u}_i^r} + \sum \frac{(\hat{\bar{m}}_i^r - \bar{m}_i^r)^2}{\hat{\bar{m}}_i^r} + \sum \frac{(\hat{v}_i^r - v_i^r)^2}{\hat{v}_i^r} + \sum \frac{(\hat{y}_i^r - y_i^r)^2}{\hat{y}_i^r} + \sum \frac{(\hat{e}_i^r - e_i^r)^2}{\hat{e}_i^r}$$

Type E table regionalization: Step 4

Balancing algorithm:
Turning previously derived estimates $\bar{z}_{ij}^r$, t_i^{sr}, u_i^r, $\bar{m}_i^r$, v_i^r, y_i^r and e_i^r into feasible estimates (without changing them too much)

Example for optimization problem with known x_i^r:

$$\text{Minimize } S = \sum \frac{(\hat{\bar{z}}_{ij}^r - \bar{z}_{ij}^r)^2}{\hat{\bar{z}}_{ij}^r} + \sum \frac{(\hat{t}_i^{sr} - t_i^{sr})^2}{\hat{t}_i^{sr}} + \sum \frac{(\hat{u}_i^r - u_i^r)^2}{\hat{u}_i^r}$$
$$+ \sum \frac{(\hat{\bar{m}}_i^r - \bar{m}_i^r)^2}{\hat{\bar{m}}_i^r} + \sum \frac{(\hat{v}_i^r - v_i^r)^2}{\hat{v}_i^r} + \sum \frac{(\hat{y}_i^r - y_i^r)^2}{\hat{y}_i^r} + \sum \frac{(\hat{e}_i^r - e_i^r)^2}{\hat{e}_i^r}$$

subject to:
$x_i^r + \sum_s \hat{t}_i^{sr} + \hat{\bar{m}}_i^r = \hat{u}_i^r$ (balance constraint)
$\sum_j \hat{\bar{z}}_{ij}^r + \hat{y}_i^r + \sum_s \hat{t}_i^{rs} + \hat{e}_i^r = \hat{u}_i^r$ (balance constraint)
$\sum_j \hat{\bar{z}}_{ji}^r + \hat{v}_i^r = x_i^r$ (balance constraint)
$\sum_r \hat{\bar{z}}_{ij}^r = \bar{z}_{ij}$; $\sum_r \hat{u}_i^r = u_i$; $\sum_r \hat{\bar{m}}_i^r = \bar{m}_i$; $\sum_r \hat{v}_i^r = v_i$; $\sum_r \hat{y}_i^r = y_i$;
$\sum_r \hat{e}_i^r = e_i$ (aggregation constraints)

Content

Concluding remarks

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Concluding remarks

Summing up:

- Constrained optimization techniques can be very useful for constructing interregional IO tables
- Initial estimates may even be infeasible, but better initial estimate lead to better final estimates
- The type of table has a significant effect on the formulation of the optimization problem
- Gravity models are useful for constructing interregional transactions in type B and type E tables

Further readings

- J. Többen T. Kronenberg (2015): Construction of multi-regional input-output tables using the CHARM method, Economic Systems Research, 27(4), 487-507.
- M. Jahn (2017): Extending the FLQ formula: a location quotient-based interregional input–output framework, Regional Studies, 51(10), 1518-1529.
- M. Jahn, A.T. Flegg, T. Tohmo (2020): Testing and implementing a new approach to estimating interregional output multipliers using input–output data for South Korean regions, Spatial Economic Analysis, Published online: 12 Feb 2020.

Reviewing non-survey approaches for constructing interregional IO tables

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